Internationally Comparable Mathematics Scores for Fourteen African Countries

Justin Sandefur

Abstract

Internationally comparable test scores play a central role in both research and policy debates on education. However, the main international testing regimes, such as PISA, TIMSS, or PIRLS, include very few low-income countries. For instance, most countries in Southern and Eastern Africa have opted instead for a regional assessment known as SACMEQ. This paper exploits an overlap between the SACMEQ and TIMSS tests—in both country coverage, and questions asked—to assesses the feasibility of constructing global learning metrics by equating regional and international scales. I compare three different equating methods and find that learning levels in this sample of African countries are consistently (a) low in absolute terms, with average pupils scoring below the fifth percentile for most developed economies; (b) significantly lower than predicted by African per capita GDP levels; and (c) converging slowly, if at all, to the rest of the world during the 2000s. While these broad patterns are robust, average performance in individual countries is quite sensitive to the method chosen to link scores. Creating test scores which are truly internationally comparable would be a global public good, requiring more concerted effort at the design stage.

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Contents

1 Introduction **2**

2 Data **8**

3 Equipercentile linking based on countries taking both tests **10**

4 Item-response theory: linking tests based on overlapping items **15**

4.1 Imposing TIMSS item parameters 19

4.2 Mean-sigma equating 27

5 Making sense of contradictory results **31**

6 Putting African mathematics scores in context **35**

7 Conclusion **38**

1 Introduction

Around the developing world, and particularly in East Africa, there is growing evidence that the expansion of primary school enrollment over the last quarter century has not delivered concomitant improvements in learning levels (Jones et al., 2014; Pritchett, 2013). The United Nation's 2015 global goals seek to address this imbalance by focusing on education quality, including indicators of proficiency in literacy and numeracy. But these indicators are not currently measurable on an international scale, particularly in Africa. Notably, only three countries in Sub-Saharan Africa have participated in any of the major international assessments of learning levels.[1]

My goal in this paper is to put mathematics test scores from an existing regional learning assessment covering fourteen African countries on an international scale using both simple statistical methods, and more formal item response theory methods. This process is known in the psychometric literature as linking or equating, terms which I use interchangeably here.[2] The regional test is the Southern and Eastern Africa Consortium for Measuring Education Quality (SACMEQ) assessment, and the international scale is provided by the Trends in International Mathematics and Science Study (TIMSS), an international assessment administered in grades three and four (population 1) and seven and eight (population 2) in over sixty countries. This linking is possible because (a) two countries, Botswana and South Africa, took both tests, and (b) the 2000 and 2007 SACMEQ rounds embedded a number of items from the TIMSS test. These overlapping items were included in the African tests with the explicit purpose of facilitating international comparisons (Ross et al., 2005, p. 71).

It appears this *ex ante* push for comparability was abandoned *ex post*. To my knowledge, no reporting of SACMEQ scores on an international scale exists in the public domain. It is widely rumored that these results were withdrawn due to the political sensitivity of highlighting the enormous learning deficiencies in all fourteen SACMEQ countries relative to the global distribution. There is mixed evidence to justify this political sensitivity. There are anecdotal reports that bench-marking student performance on international assessments has contributed to national political pressure for education reform in the OECD (Breakspear, 2012), as well as some Latin American (Bruns, 2015) and Eastern European countries

[1] Ghana has participated in PIRLS (primary-level reading assessment), and South Africa and Botswana have participated in both PIRLS and TIMSS (primary-level mathematics and science assessment).

[2] See Holland (2007) for a discussion of what makes a linking an equating; the latter generally implies greater rigor and comparability. In Hollard's terminology, this exercise might be termed a 'calibration' or 'concordance', as the pupil populations differ and the test constructs, difficulty, and reliability are not guaranteed to be identical.

(Marciniak, 2016). But experimental work in East Africa has found that dissemination of national assessments results has little effect on local political demands for education reform (Lieberman et al., 2014).

Politics aside, there are sound technical reasons to be cautious about any comparison of African learning levels to international benchmarks. When comparing populations with very different learning levels, traditional methods for test-score equating are subject to sizable non-sampling error. The size of this 'linking error' is inversely proportional to the number of overlapping items across the two the tests (Michaelides and Haertel, 2004). For instance, Hastedt and Desa (2015) present simulations using TIMSS data to show that statistically significant differences in country means may not be detected when the number of overlapping items falls below roughly thirty, as is the case here.

To address these concerns, I compare the results from three different linking approaches.

The first approach is referred to as equipercentile equating or linking in the psychometric literature (see Kolen and Brennan, 2014, chapter 4). It does not require any overlapping test items across the two tests and does not rely on item response theory to link the two test scales, above and beyond whatever IRT methods may have been used in construction of the original scores. Instead, equipercentile linking as applied here depends on the existence of data from both tests for a common population of pupils. In this case, I rely on overlapping coverage of SACMEQ (2000) and TIMSS (2003) in Botswana and South Africa, matching each percentile of the SACMEQ distribution to the corresponding percentile of the TIMSS distribution. Lee and Barro (2001), Altinok and Murseli (2007), and Altinok et al. (2014) have all applied simpler versions of this approach to link various regional and international tests, relying only on country means and variances; here I apply non-parametric methods to the full distribution and take a more conservative approach to identifying comparable populations of test-takers. Nevertheless, this procedure assumes that SACMEQ and TIMSS true scores are highly predictive of each other, and that this relationship is stable across countries. The first assumption is not testable with my data, and I find some violation of the second assumption when comparing results for Botswana and South Africa.[3]

A second, alternative approach using item-response theory relies on overlapping items across the two tests, rather than overlapping coverage in the populations tested. Das and Zajonc (2010) apply IRT methods to estimate TIMSS-equivalent scores for two states in India, and Singh (2014) applies the same procedure to regions of Ethiopia, India, Peru, and Vietnam. The two central assumptions here, as in most applications of item response theory, are unidimensionality and parameter invariance. The linking procedure implicitly assumes SACMEQ and TIMSS measure the same, singular underlying trait (which I refer to as math proficiency), and the relationship between a student's overall math proficiency and performance on any given test item is invariant across populations and demographic groups. Violations of these assumptions manifest themselves through differential item functioning (DIF), in which students with similar proficiency levels in different groups (in this case, the SACMEQ African sample versus the broader TIMSS sample) perform better or worse on a given item. While teachers in the SACMEQ sample pool quite well with the TIMSS sample, SACMEQ pupils exhibit high levels of DIF, casting some doubt on these estimates, which are considerably higher than the other two approaches – and well above the actual TIMSS scores measured for Botswana and South Africa.

A complication to this approach is that the SACMEQ pupil test includes only a few TIMSS items; however, the SACMEQ teacher test includes a longer list of TIMSS items, and the SACMEQ teacher and pupil tests also share a longer list of items. Thus I present an

[3]An alternative approach to linking international assessments that has been used and widely cited in the economics of education literature (Hanushek and Kimko, 2000; Hanushek and Woessmann, 2012) abstracts entirely from the content of the test or the distribution of pupil scores, and uses assumptions about the variance of country averages around the world to link the global distributions of various assessments. See Altinok et al. (2014) for a critique of this approach.

extension of standard linking methodologies, effectively creating a chain linkage from TIMSS to the SACMEQ teacher test and then, in turn, to the SACMEQ pupil test.

The third approach I employ also relies on item response theory, but is potentially less sensitive to DIF. This approach, known as mean-sigma equating, is commonly applied to link, e.g., subsequent rounds of testing regime. Rather than imposing all of the item level parameters from the reference population (TIMSS) on the target population (SACMEQ), it ensures only that the average level of difficulty and discrimination for the overlapping items are held constant across the two populations. Estimates based on the mean-sigma approach are largely congruent with the equipercentile method as well as the actual TIMSS scores for Botswana and South Africa.

Substantively, the results here are daunting for African education systems. Most of the national test-score averages I estimate for the thirteen African countries in my sample fall more than two standard deviations below the TIMSS average, which places them below the 5th percentile in most European, North American, and East Asian countries. In contrast, scores from the SACMEQ test administered to math teachers are much higher, but fall only modestly above the TIMSS sample average for seventh- and eighth-grade pupils, in line with earlier analysis by Spaull and van der Berg (2013). African test scores appear low relative to national GDP levels; in a regression of average scores on per capita GDP in PPP terms, average scores in the SACMEQ sample are significantly below the predicted value using all three linking methodologies. Furthermore, there is little sign that African scores were improving rapidly or converging to OECD levels during the 2000s.

A major caveat in interpreting these comparative results is that the SACMEQ test is administered to pupils in grade 6 in most countries, while TIMSS is administered to pupils in grades 7 or 8. Thus African pupils are one to two grades below their OECD counterparts when sitting these tests. However, there is some virtue to this difference. Because pupils in the African sample tend to be much older at a given grade level, the average *age* of the pupils in the SACMEQ and TIMSS data is quite similar. In most countries, the modal age in each case is fourteen.[4]

Methodologically, this exercise demonstrates the feasibility of comparing regional and international test scores on a common scale, but also highlights the fragility of this linking when relying on a very short set of anchoring items and attempting to span populations with widely disparate learning levels. Results differ across methodologies more than one might wish for drawing confident policy conclusions. When pooling original TIMSS scores with SACMEQ scores re-calibrated to the TIMSS scale using each of the three methods described here, the Spearman rank correlations of the country averages are 0.97 or higher, but the absolute level of scores varies by more than half a standard deviation in some cases. I conclude that future efforts to build a global learning metric should clearly focus on expanding the set of anchoring items across these tests.

The next section describes the SACMEQ and TIMSS data, and the overlapping items. Section 3 compares scores from the two tests without using (additional) item-response theory assumptions, based on a simple non-parametric approach known as equipercentile linking. Section 4 presents my core approach to equating the scales, extending the IRT model used to estimate TIMSS scores to the SACMEQ sample based on overlapping items. The various linking approaches do not entirely agree, and I discuss how to reconcile the results in Section 5. Section 6 steps back to examine the basic substantive findings on mathematics scores in the African sample that are relatively robust across methods. Section 7 concludes.

[4]Note that while both TIMSS and SACMEQ use grade-based sampling, other major international assessments such as PISA use age-based sampling.

2 Data

The SACMEQ project is a regional learning assessment sponsored by UNESCO's Institute for International Education Planning (IIEP). The test is administered by ministries of education in the participating countries, with technical assistance from UNESCO. The test is multiple choice and the content is uniform across countries and not tailored to national curricula. The range of competencies covered in the mathematics test are described in Hungi et al. (2010). The sample is school based and grade based, i.e., pupils are sample from those found in the sampled schools without attempting to reach children not in school, and pupils are sampled from grade 6, regardless of their age.

Data collection for SACMEQ I was completed in 1995 in seven countries, and the list of participating countries expanded to thirteen for SACMEQ II in 2000, and fourteen for SACMEQ III in 2007.[5] In round 2, which is the focus of the analysis here, the sample included roughly 150 schools per country and a total of approximately 40,000 pupils across the thirteen countries. Country-specific sample sizes are reported in Table 1.

While SACMEQ and TIMSS test different grades, the average age of the sixth-graders in the SACMEQ sample (13.8 years) is fairly similar to the average age of the seventh- and eighth-graders in the TIMSS sample (14.4), and considerably older than the TIMSS third- and fourth-graders (10.3). Note, however, that this is less true when comparing within countries in South Africa and Botswana, who participated in both SACMEQ and TIMSS (only at the eighth-grade level). In both cases, the SACMEQ sample is nearly two years younger than the TIMSS sample.

All SACMEQ scores are based on a Rasch model, with a scale standardized to have a mean of 500 and a standard deviation of 100 for the full sample in 2000. Looking across

[5] Note that the Ministry of Education of Zanzibar, a region of Tanzania, is an independent member of SACMEQ, so project documents typically refer to fourteen Ministries of Education in SACMEQ II rather than thirteen countries.

Table 1: Summary statistics before equating

	Pupils: mean (sd)		Teachers	Obs.		
	Age	Score	Score	Pupils	Teachers	Schools
SACMEQ 2000, grade 6:						
Mauritius	11.3	585		2870		147
	(.5)	(140)				
Kenya	14	562	967	3296	263	185
	(1.6)	(87)	(107)			
Seychelles	11.6	554	868	1482	44	24
	(.4)	(107)	(78)			
Mozambique	14.7	530	789	3136	308	173
	(1.9)	(56)	(99)			
Swaziland	13.8	518	804	3138	167	168
	(1.7)	(66)	(86)			
Tanzania	15.1	517	792	2849	184	181
	(1.5)	(85)	(82)			
Botswana	13.2	513	751	3321	393	170
	(1.2)	(82)	(78)			
Uganda	14.2	506	827	2619	127	162
	(1.8)	(108)	(114)			
South Africa	13.1	483		3135		167
	(1.6)	(110)				
Zanzibar	14.9	476	686	2459	162	144
	(1.6)	(63)	(84)			
Lesotho	14.1	448	737	3144	231	176
	(1.8)	(60)	(73)			
Malawi	14.6	434	770	2323	134	140
	(2.2)	(56)	(84)			
Zambia	14	432	749	2590	286	172
	(1.7)	(73)	(86)			
Namibia	13.8	431	731	4990	326	267
	(1.9)	(83)	(111)			
TIMSS 2003:						
All countries, grade 4	10.3	492		127896		3895
	(.7)	(108)				
All countries, grade 8	14.4	463		237833		7008
	(.8)	(112)				
Botswana (grade 8)	15.1	366		5150		146
	(1)	(65)				
South Africa (grade 8)	15.1	264		8952		255
	(1.3)	(100)				

The SACMEQ and TIMSS scores reported here are not comparable. The table reports sample means and standard deviations using the IRT scores as reported by SACMEQ and TIMSS, respectively, using sample weights reflecting countries' pupil populations and the survey design. Note that SACMEQ pupil and teacher scores are comparable across countries, but are not comparable to each other before equating.

countries in Table 1, the gap between the highest-scoring country (Mauritius) and the lowest scoring (Namibia) is approximately 150 points. This scale was maintained to enable trend analysis between 2000 and 2007 in SACMEQ III, during which time average math scores increased by roughly 10 points (Makuwa, 2010).

SACMEQ also administers a math test to grade 6 math teachers in sampled schools. Approximately 3,300 math teachers took this test in 2000. There are very large cross-country gaps in teacher performance in the data, with Kenyan teachers posting an average score of 967, while teachers in Zanzibar post scores nearly three standard deviations lower, with an average of just 686. Note that the teacher test contains an overlapping set of items with the pupil test, and can be placed on a common scale, but the official scores reported in Table 1 are not directly comparable between teachers and students before the equating process developed in the following sections.

Focusing exclusively on the set of common items administered to both pupils and teachers, pupils answered an average of 28% of items correctly, while teachers answered 73% correctly. Across countries, pupils' percent correct ranged from just over 20% in Malawi and Zambia to 41% in Mauritius (the latter country did not administer the teacher test). Teacher scores ranged from just 58% in Zanzibar to 91% in Kenya. The full distribution of pupil and teacher raw scores by country is shown in Figure 10 in the appendix.

3 Equipercentile linking based on countries taking both tests

Before delving into the item-level data, it is possible to compare SACMEQ and TIMSS scores at a fairly granular level while taking the official, total, score per student as given. A common approach to comparing test scales without the use of item response theory is known as equipercentile equating or linking (see Kolen and Brennan, 2014, chapter 4). This procedure is conceptually simple, but requires strong assumptions that are not testable without data on both tests from the same sample of test-takers.

The basic approach here is to match each percentile of the test score distribution on the target test (SACMEQ) to its corresponding percentile on the reference test (TIMSS). Clearly, the validity of this link hinges on (a) the two populations being tested having the same distribution of underlying proficiency, and (b) the two tests capturing the same measure of math proficiency.

To restrict estimation to comparable distributions, I focus on data from the two countries that participated in both SACMEQ and TIMSS: Botswana and South Africa. Both tests drew nationally representative populations: SACMEQ of sixth-graders in 2000 and TIMSS of eighth-graders in 2003. On the question of content, SACMEQ was designed to capture similar conceptual content to TIMSS at a broad level. Though as I show in the following sections, performance on TIMSS items in the SACMEQ sample is imperfectly correlated with overall SACMEQ performance. Nevertheless, for the purpose of the equipercentile equating in this section, I proceed under the assumption that the two tests would yield a perfect rank order correlation when applied to pupils at either grade level and at either point in time.

At a practical level, equipercentile equating consists of two basic steps. In the first step, the researcher estimates the percentiles of the target and reference tests. Here I use one hundred raw percentiles from each distribution. (An alternative approach common in the psychometric literature, is to use one of various methods to 'pre-smooth' these densities.) After equating the percentiles from the respective tests, the second step is to estimate a continuous relationship between the two score distributions, known as post-smoothing. I use a cubic spline function with knots at the score deciles to post-smooth the SACMEQ-TIMSS score relationship. The main purpose of post-smoothing in this application is interpolation, to provide a correspondence for test scores not observed in the two country samples used for equating.

Turning to the results, there is some cause for concern about the adequacy of equipercentile linking in this context, and I stress that the results here are intended to be illustrative of the method, rather than providing a definitive linking.

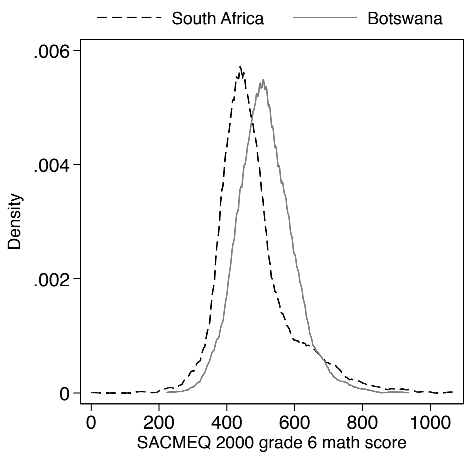
While Botswanan students score more than a quarter of a standard deviation higher on SACMEQ than South African students – 513 versus 486 on average – the kernel densities for the two countries show that their score distributions overlap considerably (Figure 1a). The performance gap between the countries appears considerably larger on TIMSS, where Botswana's average score is a full one-hundred points higher than South Africa's: 366 versus 263 on average. The South African average falls below the fifth percentile on the Botswana distribution (Figure 1b). (Interestingly, South Africa outperforms Botswana at the very top of the distribution in both tests, consistent with South Africa's highly unequal education system.)

The fact that relative country performance across most of the distribution differs so dramatically on the two tests suggests that link between SACMEQ and TIMSS scores is unlikely to be stable across the two countries. This is indeed what Figure 1c shows. The figure equates the percentiles of the SACMEQ distribution to the corresponding percentile of the TIMSS distribution for each country, as well as a combined distribution that gives equal weight to observations from each country. The circles and crosses on the graph denote raw percentiles, while the gray lines show a local polynomial fit of the relationship between TIMSS and SACMEQ percentiles (known as "post-smoothing" in the psychometric literature).

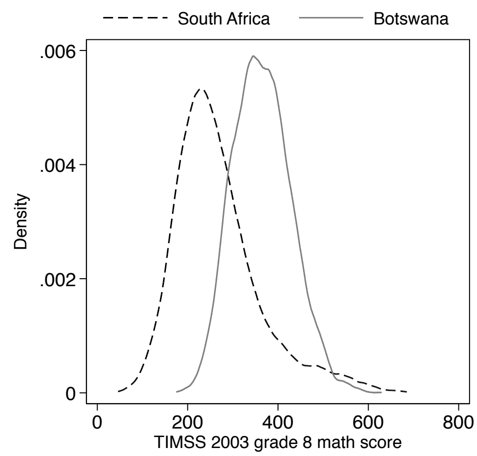
In Botswana, a SACMEQ score of 400 is just below the eighth percentile, which is equivalent to a 279 on the TIMSS scale. In South Africa, the same SACMEQ score would be in the twentieth percentile, which is equivalent to a score between 180 and 190 on TIMSS. So

Figure 1: Equipercentile equating

(a) Kernel densities for SACMEQ



(b) Kernel densities for TIMSS



(c) Matching raw percentiles (and post-smoothing)

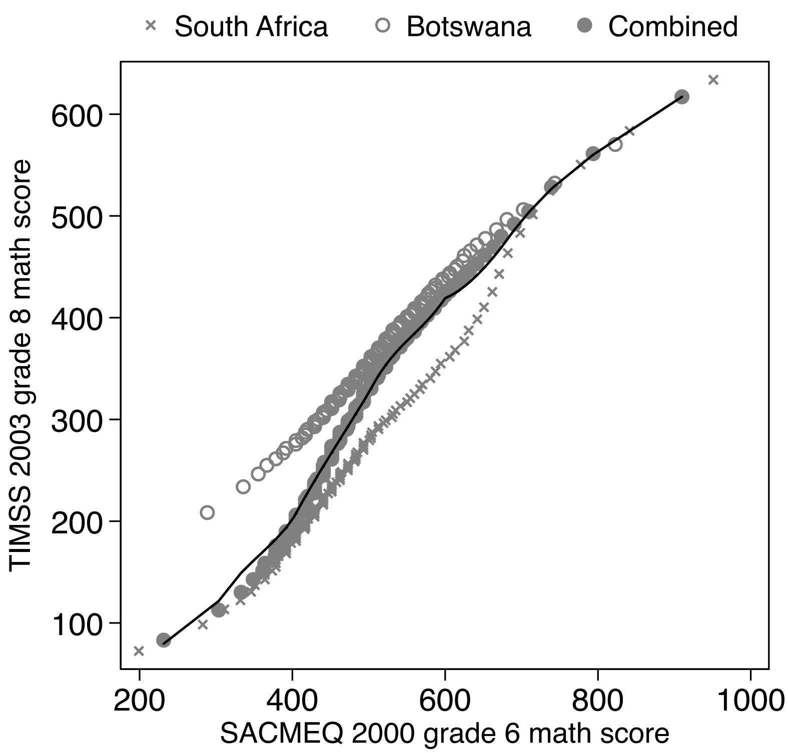
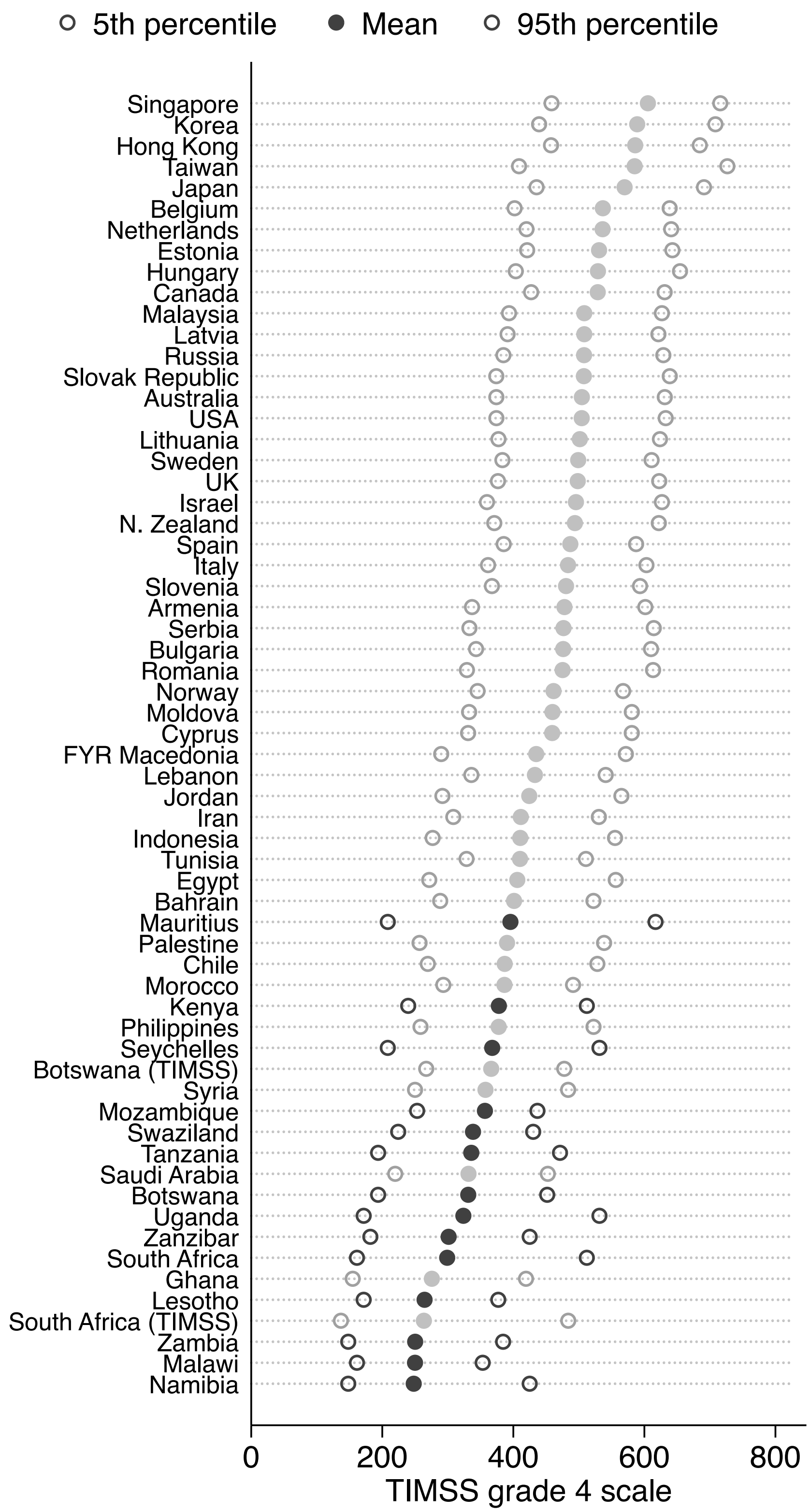


Figure 2: Country rankings: TIMSS grade 8 scale, equipercentile linking



the predicted TIMSS score for children with a given SACMEQ score differs by roughly a full standard deviation, depending on which country is used to make the link.

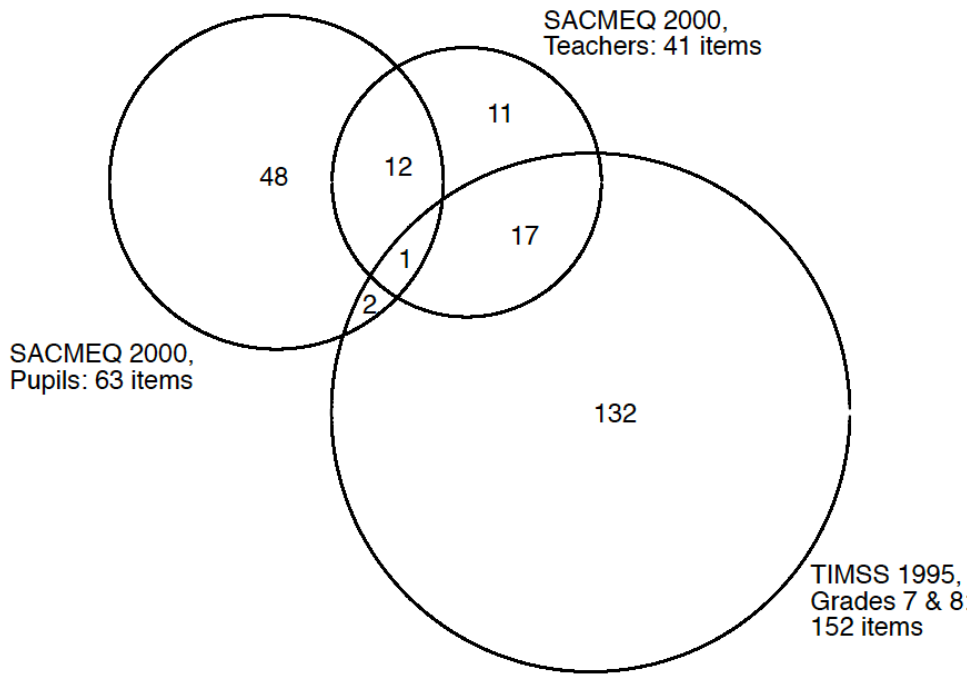
The reliability of the link varies across the score distribution, as seen in Figure 1c. The gap between the Botswana and South Africa results is large at the lower end of the scale and remains large up until a SACMEQ score of approximately 600 (around the 85th percentile in both countries); at the very top of the distribution the gap narrows considerably, with a discrepancy of roughly fifteen points in predicted TIMSS scores (500 versus 485) for a SACMEQ score of 700 (the 98th percentile in Botswana and the 95th in South Africa).

Setting aside these methodological concerns, equipercentile linking places the African countries in SACMEQ firmly at the bottom of the TIMSS country rankings. (See Figure 2.) All fourteen countries post average scores with a TIMSS equivalent of less than 400, and in some cases dramatically lower (e.g., Zambia, Malawi, and Namibia at or below 250). Based on this simple equating method, the fifteen African countries participating in either test occupy fifteen of the bottom twenty places on the country ranking in Figure 2, as well as nine of the bottom ten slots.

4 Item-response theory: linking tests based on overlapping items

Both the SACMEQ and TIMSS tests are built on an item response theory (IRT) framework, which generates test scores by modeling the probability a given pupil answers a given test item correctly as a function of pupil- and item-specific characteristics. In principle, the same core assumptions required to estimate these IRT models for a given test also make it possible to apply the item parameters to a new population and generate comparable scores for pupils who sit the same test items.

Figure 3: Heuristic of overlapping math items



The Venn diagram shows the overlapping or "anchor" items linking the SACMEQ 2000 pupil and teacher tests to the TIMSS 1995 test of seventh- and eighth-graders.

For the purposes of this paper, a key feature of SACMEQ II is the inclusion of a sub-set of items taken from the TIMSS 1995 round. This overlap is documented in an appendix to a methodological chapter written by the core SACMEQ team and reproduced in each of the national reports, e.g., Ross et al. (2005).[6] I provide a heuristic depiction of the SACMEQ-TIMSS overlap in Figure 3. The SACMEQ pupil test embeds six anchoring items from the TIMSS grade 3 and 4 test, and just three items from the TIMSS grade 7 and 8 test. The SACMEQ teacher test serves as an additional link, however, between the SACMEQ and TIMSS scales. The teacher test contains thirteen items from the SACMEQ pupil test and eighteen items from the TIMSS grade 7 and 8 test. Together, these overlapping items form the basis of the IRT linking methodology described below.

[6] I'm grateful to Nic Spaull for pointing me to this appendix.

Even without any recourse to item response theory, the raw percentage-correct scores on these overlapping items shows that SACMEQ teachers perform somewhat better than seventh- and eighth-graders in the TIMSS sample, 58% to 41%, and unsurprisingly, that SACMEQ teachers perform much better than their pupils, 76% to 29%. (Out of the seventeen overlapping questions between SACMEQ teachers and TIMSS pupils, there is just one exception where the latter perform better, and on the thirteen overlapping questions between SACMEQ teachers and pupils, there are no exceptions. See Table 2.) The core of the IRT linking methodology is to connect those two ratios, though instead of percentage correct, I rely on parameters from a conditional logit model.

Some direct comparisons between TIMSS and SACMEQ pupils are also possible, without passing through the teacher test. While I focus here on the TIMSS grade 7 and 8 sample, the grade 3 and 4 test has more direct overlap with SACMEQ. TIMSS pupil performance strictly dominates SACMEQ pupils on all overlapping questions, despite the latter being on average four years older.

The correct answer percentages are also informative about the relative difficulty of the different tests. One might expect that in designing the SACMEQ test, researchers would complement the TIMSS items with additional items that were considerably easier, to extend the TIMSS scale on the low end for a context where learning levels are anticipated to be low. This did not happen to any great extent. On average SACMEQ pupils answered 32% of the TIMSS items correctly and 39% of the non-TIMSS items correctly, implying that the items unique to SACMEQ were not dramatically easier.[7]

In the following sub-sections I present two IRT-based methods for linking SACMEQ scores to the TIMSS scale using these overlapping items. Both approaches make use of the published item-level parameters from the IRT model underlying the TIMSS scale, combined

[7] These numbers differ from the numbers in Table 2 because they include TIMSS grade 3 and 4 items that appeared on the SACMEQ pupil test.

Table 2: Anchoring items: percent correct

Item ID	TIMSS Pupils (1)	SACMEQ Teachers (2)	SACMEQ Pupils (3)
tmath04	61.1	94.1	
tmath09	53.7	63.7	
tmath17	41.1	40.4	
tmath18	43.4	56.1	
tmath19	41.6	46.8	
tmath20	38.4	55.9	
tmath22	38.9	73.7	
tmath23	32.2	35.9	
tmath24	31.8	59.1	
tmath25	22.7	48.6	
tmath26	47.1	78.4	
tmath27	33.8	63.2	
tmath29	41.8	64.1	
tmath30	44.3	50.7	
tmath31	40.1	60.6	
tmath34	40.8	44.2	
pmath63	38.6	48.6	22.8
pmath26		41.9	18.6
pmath27		92.0	30.4
pmath28		82.5	42.2
pmath29		78.6	50.2
pmath30		69.3	22.6
pmath32		85.0	20.1
pmath33		81.7	18.4
pmath44		86.5	27.8
pmath55		50.8	16.8
pmath56		92.5	28.1
pmath57		82.9	24.4
pmath58		92.0	51.5
pmath47	86.8		36.3
pmath50	84.5		36.8
Average on overlap			
TIMSS to SACMEQ teachers	40.7	57.9	
SACMEQ teachers to pupils		75.7	28.8

Note that one of the 17 items shown in Figure 3 in the overlap between TIMSS and the SACMEQ teacher test is not listed here. It is a partial credit item dropped from the 3PL IRT model used in the analysis.

with new estimates of the TIMSS IRT model using the SACMEQ data. In the first approach, the item-level parameter estimates from the TIMSS sample are imposed as constraints when performing estimation with the SACMEQ data, following the procedure outlined by Das and Zajonc (2010). In the second approach, I estimate item-level parameters from the SACMEQ data without imposing these constraints, then rescale the parameters ex post so that the average difficulty and discrimination parameters for the overlapping items match the TIMSS estimates.

In both methods, the linking requires two steps: first to put SACMEQ teacher scores on the TIMSS scale, then to make SACMEQ pupil scores comparable to the re-scaled teacher scores.

4.1 Imposing TIMSS item parameters

Taking a step back, TIMSS items differ in their difficulty and in their discrimination. Because not all students answer the same set of questions, TIMSS relies on item response theory (IRT) methods to estimate the item-specific and student-specific components of the score. This requires specification of an item-response function (IRF). The parameters of the IRF are estimated on a sub-set of TIMSS students, known as the calibration sample, and are then applied to all students to create scaled scores.

Here I offer only a brief summary of the scaling methodology used in TIMSS; for a full exposition see Gonzalez et al. (2004). SACMEQ II drew items from the original 1995 TIMSS round. From 1999 onward, TIMSS has used an IRF based on a three-parameter (3PL) logistic model, which was applied retrospectively to the 1995 data as well. Let $x_{ij} \in \{0, 1\}$ denote the response of individual i to item j, where 0 indicates an incorrect response and 1 a correct response. The IRF gives the probability that i answers a given item correctly conditional on her mathematics proficiency, θ_i, and the item's discrimination, a_j (i.e., how quickly the probability of a correct answer increases as overall math proficiency increases), difficulty, b_j, and the probability of guessing correctly, c_j.

$$\Pr(x_{ij} = 1 | a_j, b_j, c_j, \theta_i) = c_j + (1 - c_j) \frac{\exp\{a_i(\theta_i - b_j)\}}{1 + \exp\{a_j(\theta_i - b_j)\}}. \tag{1}$$

As an aside on terminology, the latent variable θ is often referred to as "ability" in the psychometric literature, whereas labor economists typically reserve the term ability for an innate, unteachable, and often unobservable form of intelligence. To avoid confusion, I refer to θ as mathematics proficiency, or just proficiency. There is no suggestion here that proficiency is innate. Rather, it is a measure of a pupil's learning level as determined by school quality and other influences. The variable is latent in the sense that a pupil's true proficiency is a population parameter, which one might think of as their true score across infinite attempts on a population of infinite test items on the TIMSS scale.

Parameter estimates for equation (1) for the grade 7 and 8 TIMSS 1995 items were published in an appendix to the 1999 technical report (Martin et al., 2000). I use these estimates to link SACMEQ scores to the TIMSS grade 7 and 8 scale used from 1999 onward. Unfortunately, it is not possible to do this for the grade 3 and 4 test, which I can only link to the 1995 scale.[8] Because linking for these lower grades relies on a smaller set of overlapping items, is restricted to a one-parameter Rasch model, and uses a TIMSS scale that was abandoned in the 2000s, I relegate this analysis to an appendix.

Rather than use existing SACMEQ scores, I estimate them using equation (1) and the item-level micro data.[9]

[8]When testing of fourth-graders resumed in 2003, this scale was also updated, and 3PL estimates for the fourth-grade items were published in the 2003 technical report (Mullis et al., 2004). The unique identifiers attached to the items were changed in 1999. The TIMSS codebooks from 1999 provide a link to the original 1995 item codes for grades 7 and 8, but the 2003 codebooks do not provide a similar link for grades 3 and 4.

[9]This is necessary for two reasons. First, to my knowledge, the item parameters for SACMEQ are not publicly available. The publicly released data does include IRT-based scores (i.e., pupil-level estimates of

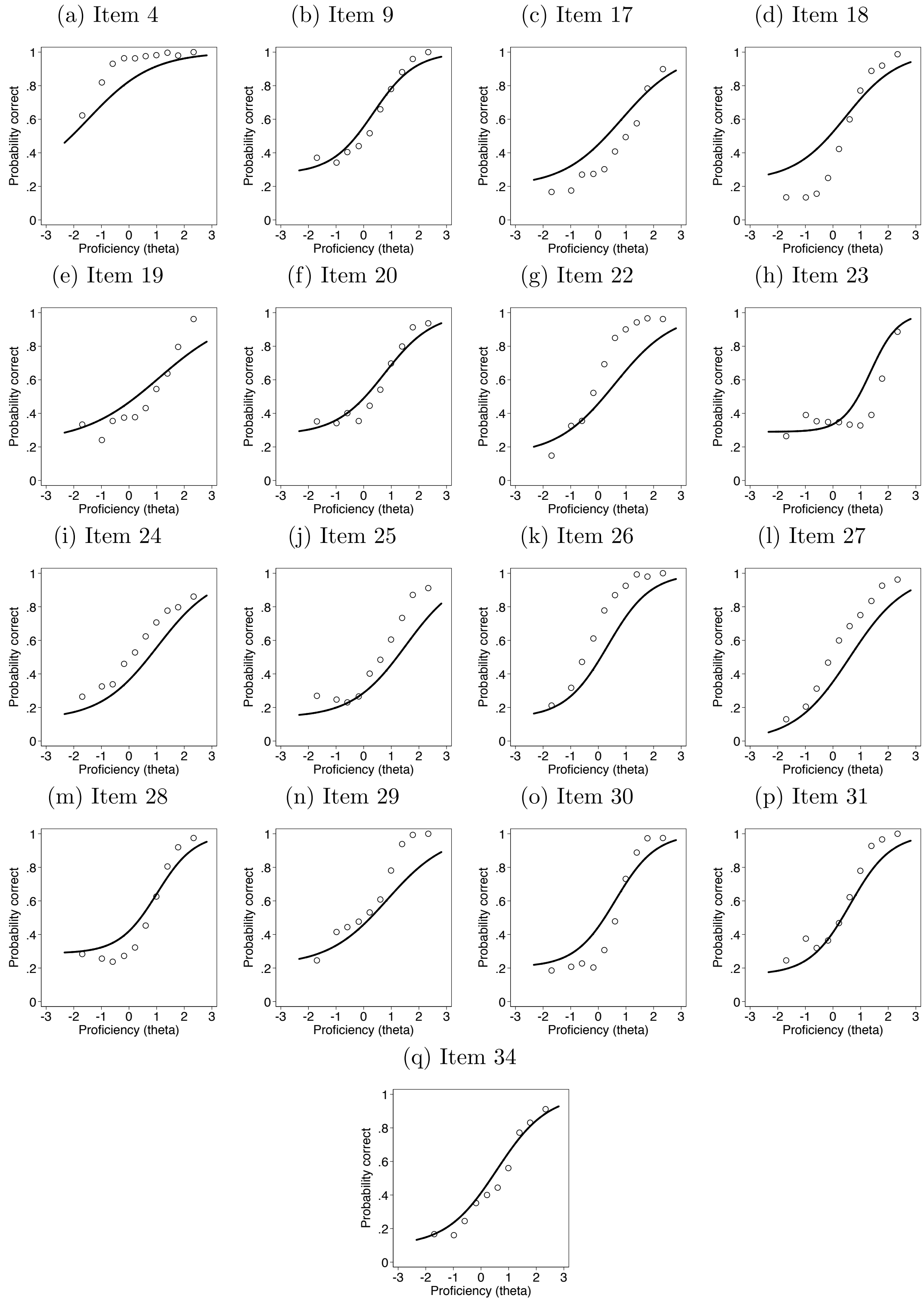
Das and Zajonc (2010) offer a clear exposition of the challenges to estimating IRT pupil parameters based on a relatively short list of test items. The central issue they highlight is that standard maximum likelihood estimates of θ_i proficiency parameters in equation (1) will overstate the variance of the test score distribution, combining true variance and measurement error. An alternative approach based on Bayesian methods (EAP) will tend to understate the true test score variance. Intuitively, a short list of items provides little information to update the posterior about the proficiency of any given individual and instead estimates are clustered near the prior. Mislevy et al. (1992) proposed a method to overcome this challenge, by drawing a sample of 'plausible values' for student i's true test score from the posterior estimate of θ_i. Das and Zajonc (2010) outline a method of estimating these plausible values using a Markov Chain Monte Carlo (MCMC) method, and provide code to implement this approach in Stata (Zajonc, 2009), which I employ here. These plausible values are used to analyze the test score distributions in the tables and figures that follow.

Estimation proceeds in two steps. In the first step, I estimate equation 1 using the SACMEQ teacher data, holding parameters on the overlapping TIMSS items to match the published TIMSS estimates. In the second step, I estimate equation 1 again, this time using the SACMEQ *pupil* data, holding constant the parameters on the overlapping items from the teacher test. Estimation at each step yields estimates of the latent variable, math proficiency, for individual teachers and pupils, respectively.

Item-characteristic curves (ICC) based on the estimated item-level parameters from equation (1) are shown in Figures 4 (for the TIMSS-SACMEQ teacher link) and Figure 5 (for the SACMEQ teacher-pupil link) for the overlapping items. Lines show the estimated ICC

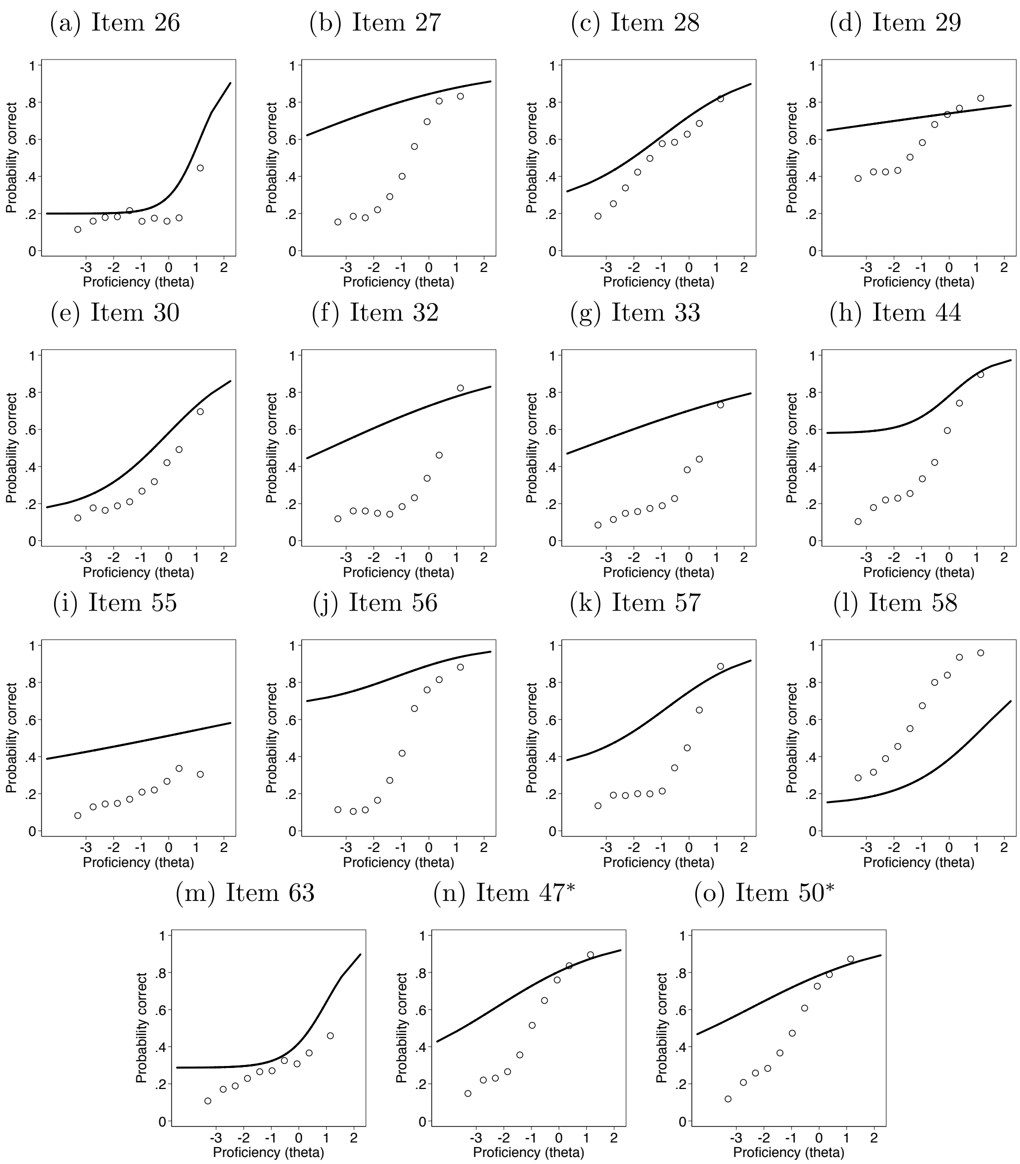
θ_i), which according to the SACMEQ reports are based on a one-parameter Rasch model (equivalent to imposing $a = 1$ and $c = 0$ for all j in equation (1)). I can reproduce these published SACMEQ scores by estimating a Rasch model with the SACMEQ item-level microdata, yielding a correlation of 0.99 at the pupil level. Second, for comparison with TIMSS scores it is preferable to apply the more flexible three-parameter model that underlies the contemporary TIMSS scale.

Figure 4: DIF: SACMEQ teachers vs. IRF from TIMSS 7th- and 8th-graders



Note: Dots represent the observed percent correct for SACMEQ teachers. Lines show the predicted percent correct based on the estimated IRF for TIMSS pupils in grades 7 and 8.

Figure 5: DIF: SACMEQ pupils vs. IRF from SACMEQ teachers



Note: Dots represent the observed percent correct for SACMEQ pupils. Lines show the predicted percent correct based on the estimated IRF for SACMEQ teachers.

* Items 47 and 50 are not on the teacher test, but are included in both the TIMSS and SACMEQ pupil tests. The IRF shown is based on the TIMSS parameters.

based on the fixed parameters from the reference population and circles show the average percent correct from the 'target' or 'focal' population.

These figures address the central question in evaluating any IRT modeling exercise, namely whether the TIMSS parameters produce a reasonable fit when applied to SACMEQ teacher data, and in turn, whether SACMEQ teacher parameters produce a reasonable fit when applied to SACMEQ pupil data. The standard approach to this question is to test for differential item functioning (DIF). If the TIMSS parameters from equation (1) are appropriate for the SACMEQ teacher data, then, e.g., a TIMSS pupil and a SACMEQ teacher with the same estimated proficiency level should have the same probability of answering a given item correctly, $P_{timss}(\theta) = P_{sacmeq}(\theta)$. If this condition fails to hold, the underlying assumptions of the IRT model are cast in doubt.

Standard approaches to exploring DIF, such as the Mantel-Haenszel (MH) test statistic, are not feasible in this context, as they require item-level data for both the reference (e.g., TIMSS) and focus (SACMEQ) groups, and the former are not available here.[10] Instead, I rely on a simple area metric similar to that originally proposed by Raju (1988), measuring the distance between the item-characteristic curve (ICC) for the reference population and the actual responses for the focal group.

$$Area = \sum_{k \in ref, foc} \Delta\theta_k |P_{ref}(\theta) - P_{foc}(\theta)| \tag{2}$$

Somewhat surprisingly, DIF is a much greater problem in the second linking step than the first: it appears more feasible to link SACMEQ teachers to TIMSS pupils than SACMEQ pupils. This is immediately evident in comparing the graphs in Figures 4 and 5. The area

[10] I am able to perform MH tests for some grade 3 and 4 TIMSS items, as reported in Table 6 in the appendix. As noted above, TIMSS does publish item level data. However, the required item-level identifiers are not available to link the TIMSS item-level data to the published 3PL parameters for the grade 7 and 8 scale for the 1995 round. An additional complication is that the TIMSS grade 7 and 8 items used in SACMEQ were spread across booklets and never administered jointly to the same pupil in the TIMSS sample.

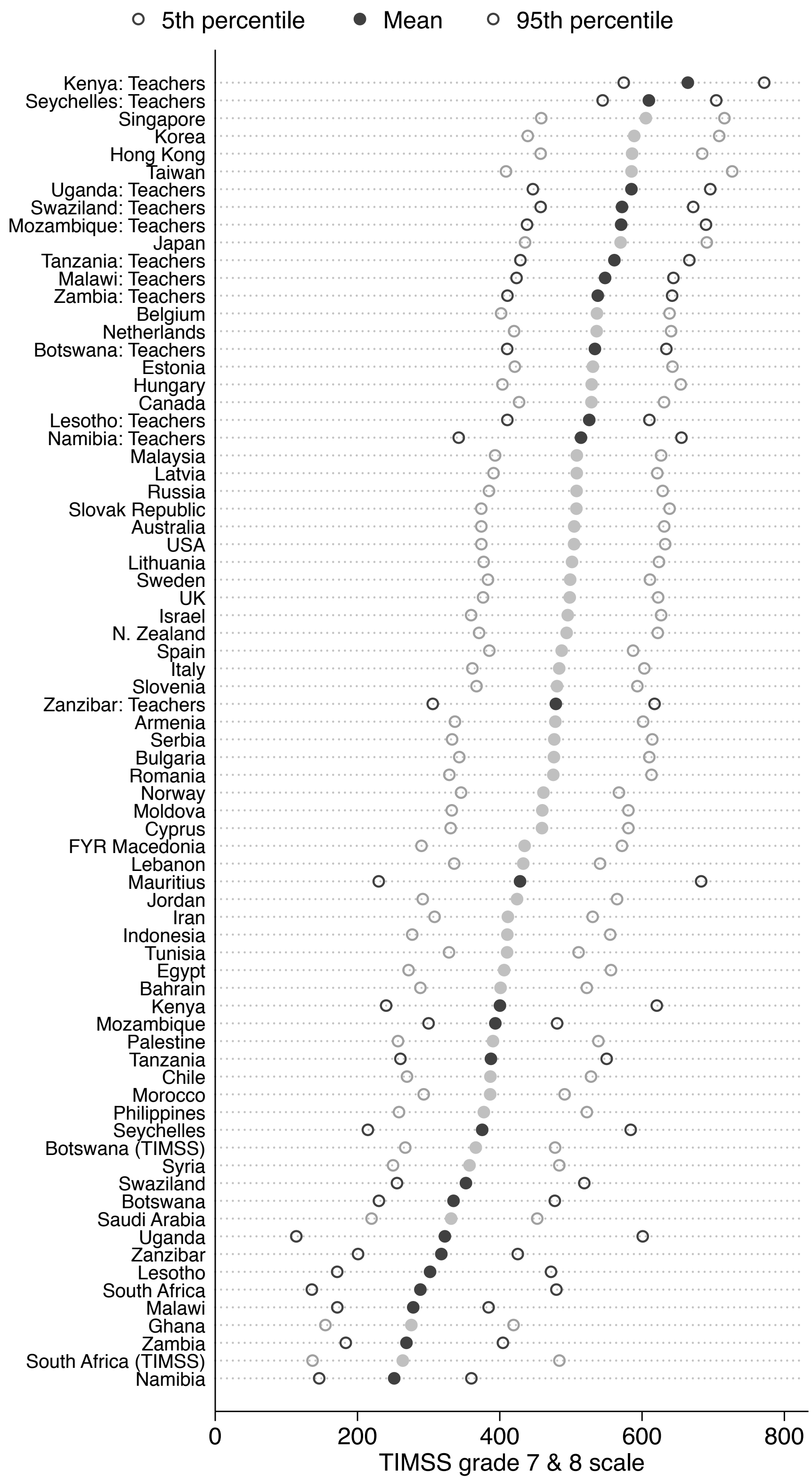
measure from equation (2) ranges from a low of 0.48 to a high of 0.78 across the overlapping items linking the TIMSS test to SACMEQ teacher test, and from 0.49 all the way up to 1.89 for SACMEQ teacher-pupil anchor items. This high level of DIF in the second step is fairly constant across countries, averaging 1.10 across items in the lowest-DIF country (Zimbabwe) and 1.37 in the highest-DIF country (Malawi).

Overall, this high level of DIF casts doubt on the reliability of this direct IRT-linking approach. The country rankings based on this link are shown in Figure 6. The African sample scores modestly higher than in the equipercentile linking in the previous section. Mauritius and Kenya score around 400, while all other African countries in the sample score lower, in some cases dramatically so, e.g., Namibia scoring just over 250 on average. (For comparison, using a similar methodology, Das and Zajonc (2010) find mean scores near 400 on the TIMSS scale for two Indian states, Orissa and Rajasthan, with higher variance than most countries in the original TIMSS sample. The latter also holds here.)

To explore the source of the DIF between pupils and teachers in the SACMEQ sample, I re-estimated the θ parameters and item-response functions two alternative ways. First, I dropped from the set of anchoring items the four items with the highest DIF from the link between the teacher and pupil test (items 27, 32, 33, and 56). This reduced overall DIF by approximately 20%, somewhat mechanically by excluding the items with the highest DIF from the calculation. However, there was no noticeable change in the country mean scores or rankings. Second, I repeated all the estimation separately for each of the African countries in the sample. This produced only a very minor reduction in DIF. Results from both exercises are excluded for brevity, but available upon request.

In the next section, I examine the plausibility of these IRT estimates more carefully. Before turning to that comparison though, I present a less restrictive approach to IRT linking that may be less sensitive to DIF.

Figure 6: Country rankings: IRT linking using TIMSS item parameters



4.2 Mean-sigma equating

The conditional logit parameters from standard IRT models such as equation (1) have an indeterminate scale up to any linear transformation. Comparing estimates of parameters from overlapping items resolves this indeterminacy. The previous section imposed the constraint that these parameters be identical, item by item. Here I rely on a less restrictive method that benchmarks the average difficulty and discrimination of the overlapping items as a whole. This procedure, known as "mean-sigma" equating (Kolen and Brennan, 2014) in the psychometric literature, produces a single linear transformation to be applied to the non-overlapping items.[11]

I begin by estimating equation (1) separately for SACMEQ teachers and pupils, without imposing parameter constraints. The estimated item-level parameters for the overlapping parameters are reported in Table 3.

Based on the invariance assumptions at the core of item-response theory (Lord, 1980), true scores on any two equivalent tests should be related by a linear transformation, such that $\theta_t = A \times \theta_r + B$, where subscripts r and t denote the 'reference' and 'target' tests, respectively. (In this case, TIMSS is the reference test and SACMEQ is the target test.) This same linear transformation applies to the parameters from the IRF:

$$a_{jr} = \frac{a_{jt}}{A_{rt}} \tag{3}$$

$$b_{jr} = A_{rt} b_{jt} + B_{rt} \tag{4}$$

In theory, a single A and B parameter should solve equations (3) and (4) for every item in the two tests. With overlapping items, this is verifiable and usually false due to measurement

[11] More flexible alternatives also exist, based on the test characteristic curves rather than first and second moments of the item parameters, and future revisions may benefit from exploring these methods.

Table 3: Mean-sigma equating: Three-parameter IRT model estimates

	TIMSS Pupils			SACMEQ Teachers			SACMEQ Pupils		
	a_i	b_i	c_i	a_i	b_i	c_i	a_i	b_i	c_i
Item	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
M012139	0.86	-1.48	0.20	0.50	-3.14	0.58			
M012085	1.29	0.34	0.14	0.63	-1.52	0.12			
M012072	0.95	0.60	0.15	0.78	-0.67	0.30			
M012123	0.93	0.86	0.22	0.42	-0.54	0.20			
M012074	0.96	1.03	0.13	0.33	-0.23	0.16			
M012113	1.31	0.35	0.27	0.59	0.03	0.31			
M012134	1.37	0.62	0.20	0.81	0.06	0.08			
M012062	1.06	0.47	0.23	0.86	0.11	0.18			
M012136	1.35	0.62	0.16	1.05	0.40	0.38			
M012091	0.99	0.61	0.00	0.63	0.52	0.40			
M012070	1.14	0.75	0.27	0.72	0.61	0.32			
M012082	1.03	1.53	0.14	0.67	0.63	0.22			
M012150	1.09	0.57	0.10	0.48	0.68	0.17			
M012066	0.73	1.10	0.23	0.56	1.03	0.26			
M012056	0.94	0.84	0.20	0.34	1.03	0.08			
M012079	1.98	1.36	0.29	0.46	3.02	0.30			
M012101	1.46	1.01	0.29	0.87	0.65	0.24	0.27	4.12	0.11
pmath29				0.14	-4.36	0.27	0.36	0.64	0.19
pmath27				0.43	-4.30	0.04	0.60	1.68	0.12
pmath56				0.47	-2.85	0.56	1.07	1.46	0.14
pmath28				0.45	-2.08	0.24	0.41	1.28	0.19
pmath44				0.59	-1.70	0.34	0.78	1.82	0.16
pmath33				0.88	-1.47	0.04	0.77	2.78	0.12
pmath57				0.74	-1.27	0.36	0.38	3.17	0.11
pmath32				1.22	-1.06	0.32	0.58	3.25	0.12
pmath30				0.60	-0.42	0.29	0.21	5.16	0.09
pmath26				0.92	0.60	0.13	0.82	4.14	0.17
pmath58				0.67	0.63	0.14	0.39	-0.23	0.02
pmath55				0.13	1.51	0.16	0.38	4.84	0.13
TIMSS-SACMEQ Teachers: μ_b		0.657			0.137				
TIMSS-SACMEQ Teachers: σ_b		0.623			1.338				
		$A = 0.465,\ B = 0.592$							
SACMEQ Teachers-Pupils: μ_b					-0.399			2.754	
SACMEQ Teachers-Pupils: σ_b					1.531			1.601	
					$A = 0.956,\ B = -3.033$				
		$A = 0.445,\ B = -2.465$							

Columns (1)-(3) report estimates using the TIMSS data reproduced from the TIMSS technical report (Martin et al., 2000). The remainder of the table reports the author's Bayesian (EAP) estimates of the item parameters for SACMEQ from a conditional logit (3PL) model. Note that while esimation uses all items from each test, the table only reports parameters for items used in linking. Bottom rows report parameters for mean-sigma equating, as described in the text.

error and an imperfect fit of the model. Thus researchers are forced to select a compromise among the various values of A and B that would solve these equations for each item. One of the simpler methods, known as the 'mean/sigma' approach, takes the mean and standard deviation of the b values in equation (4), which produces the following expressions:

$$A_{rt} = \sigma(b_r)/\sigma(b_t) \tag{5}$$

and

$$B_{rt} = \mu(b_r) - A_{rt}\mu(b_t) \tag{6}$$

where $\mu(.)$ and $\sigma(.)$ represent the mean and standard deviations of the respective parameters, taken over the set of overlapping items.

In the current application, I wish to transform SACMEQ pupil scores to the TIMSS scale, via the SACMEQ teacher scale. Refer to the intermediate exam, or 'linking' exam, by the subscript L. Applying equations (5) and (6) iteratively yields the following linear transformations:

$$A'_{rt} = A_{rl}A_{lt} \tag{7}$$

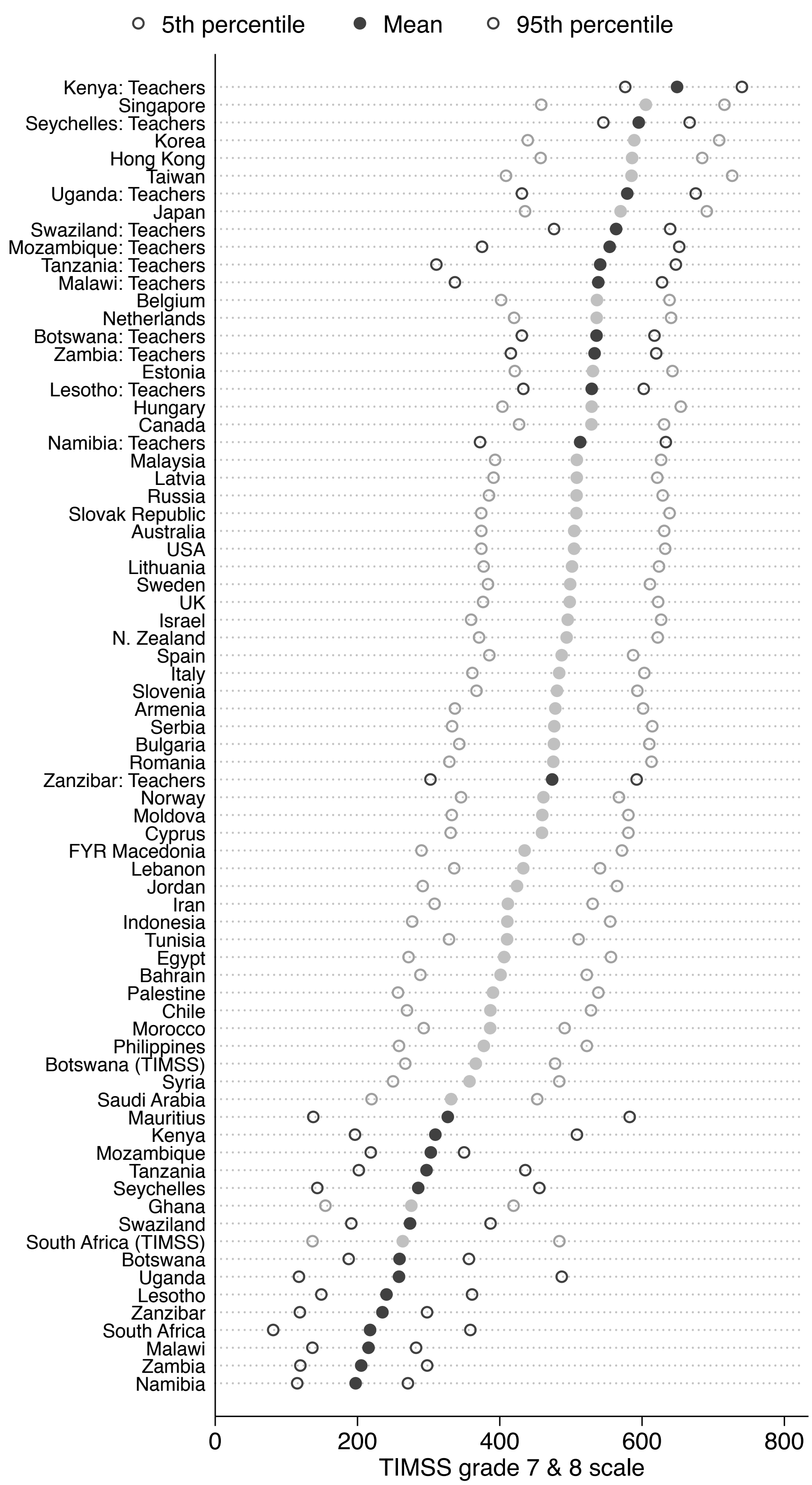
and

$$B'_{rt} = A_{lt}B_{rl} + B_{lt} \tag{8}$$

Note that the rl and lt parameters are estimated over different but potentially overlapping sets of items. Specifically, the rl parameters are estimated using the eighteen items shared between the SACMEQ teacher and TIMSS tests in Figure 3, while the lt parameters are estimated using the thirteen items shared between the SACMEQ pupil and teacher tests.

These transformation coefficients can be applied directly to the pupil-level proficiency parameters, θ_i, extracted from the 3PL model in Table 3 to convert SACMEQ scores to the TIMSS scale.

Figure 7: Country rankings: 'mean-sigma' IRT linking



The bottom of Table 3 shows the parameters for this calculation. For instance, the measured difficulty for the same item is, on average, somewhat higher for TIMSS pupils (0.657) than for SACMEQ teachers (0.137), though the variance across items is significantly higher for the teachers. These statistics generate values of $A_{\text{TIMSS,SACMEQ teacher}} = 0.465$ and $B_{\text{TIMSS,SACMEQ teacher}} = 0.592$. The same procedure applies to the second link, from SACMEQ teachers to pupils, where average difficulty parameters are dramatically higher for the latter group (-0.399 versus 2.754). In total, these parameters combine to yield transformation coefficients of $A_{\text{TIMSS,SACMEQ pupil}} = 0.445$ and $B_{\text{TIMSS,SACMEQ pupil}} = -2.465$.

The final results of this IRT-based linking are mostly congruent with the simpler equipercentile linking in the previous section. Ranking countries by their average score in the 2003 TIMSS round, the countries in the SACMEQ sample are at the very bottom of the league table in Figure 7. The only countries with comparable performance are Ghana and Saudi Arabia.

5 Making sense of contradictory results

Do the results from the three different approaches to linking African countries' mathematics scores to the TIMSS scale produce a consistent picture? Comparing countries, the correlation between the country-year averages is above 0.9 in each case, but at the pupil level it falls below 0.9 in two of three cases (see Table 4). Looking at the score distribution for each country using each of the three linking methodologies highlights the disagreements, and suggests the mean-sigma results may be the outlier. (See Figure 11 in the appendix.)

It is informative to compare not just three, but four sets of results that are available for Botswana and South Africa: the validity of the pure IRT and mean-sigma results using IRT methods and overlapping items can potentially be adjudicated by examining how well they correspond to the equipercentile results based on overlapping population coverage and the original TIMSS scores for these countries.

The pure IRT results in Section 4.1 appear to be an overestimate of the original TIMSS results, while the mean-sigma linking is an underestimate. There is no clear pattern for the equipercentile results. For instance, South African eighth-graders scored an average of 263 on TIMSS in 2003. The equipercentile method yields a TIMSS-equivalent score for South African sixth-graders of 299 in 2000. In contrast, the pure IRT link in Section 4.1 suggests South Africa's TIMSS equivalent score on SACMEQ was 317. Note that the equipercentile method automatically adjusts for the fact that sixth-graders should be expected to score lower; inasmuch as the whole method is valid, these scores are comparable. The IRT methods do not adjust for differences in the population, so we should expect *lower* scores on these measures. This is what the mean-sigma approach produces, with a mean score of 218 for South Africa. This suggests the pure IRT results are implausibly high, possibly due to the high level of DIF detected in the item-level data.

The results for Botswana give some credence to the fixed-parameter linking, despite the problematic DIF results. Botswana's eighth-graders scored an average of 366 on TIMSS in 2003 and the equipercentile method yields a TIMSS-equivalent score for Botswana's sixth-graders of 340 in 2000. Comparing the two IRT results, however, the pure IRT link in Section 4.1 yields a TIMSS equivalent of 347 while the mean-sigma approach in this section yields a score of just 260. The fixed-parameter approach is only an overestimate of the original TIMSS score if we assume that students in Botswana improve their TIMSS performance by much more than twenty points between sixth grade and eighth grade – not an impossible scenario, but not patently obvious either.

Finally, a common problem across all of the linking or equating methods used here is that the underlying test content from TIMSS may be poorly suited for the countries in question,

Figure 8: African countries' mean scores relative to the Test Information Function (TIF) for TIMSS

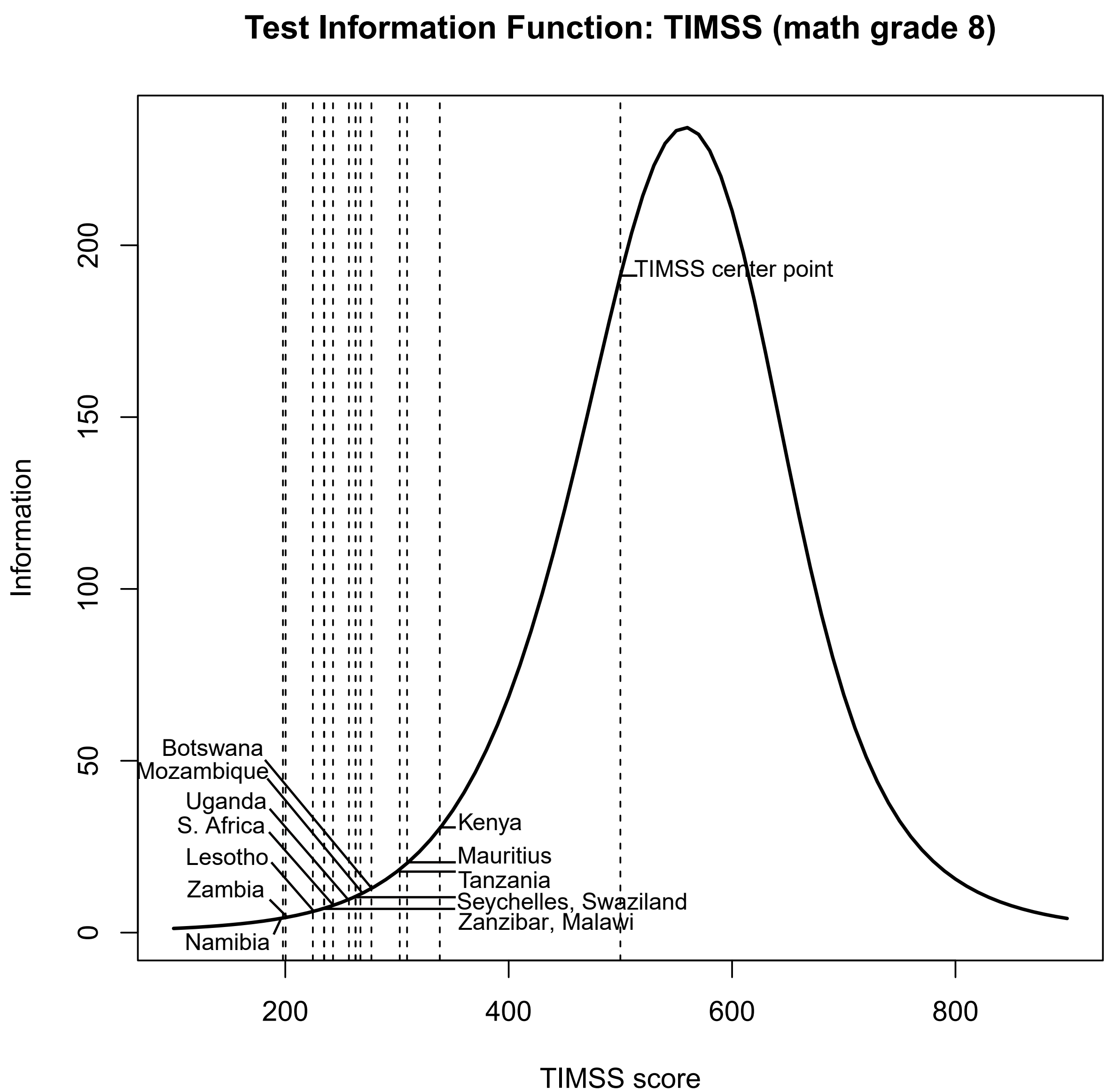


Table 4: Correlation matrices for results based on three linking methods

Variables	Equipercentile	Fixed parameters	Mean-sigma
Pupil-level data			
Equipercentile	1.00		
Fixed parameters	0.90	1.00	
Mean-sigma	0.88	0.84	1.00
Country-year averages			
Equipercentile	1.00		
Fixed parameters	0.97	1.00	
Mean-sigma	0.98	0.91	1.00

The table shows two separate correlation matrices. In both cases, all three variables are measures of pupil mathematics proficiency; they differ only in the methodology used to convert SACMEQ data to the TIMSS scale, as described in the text.

given the mathematics proficiency levels of pupils. In short, the test may simply be too hard. To see this, Figure 8 overlays the average scores for each African country in the sample (using the mean-sigma linking results) on top of the test information function (TIF) for the TIMSS grade 7 and 8 scale.[12] The TIF illustrates where, in the proficiency distribution, a test is able to distinguish student proficiency. Tests comprised of items that are uniformly very difficult may distinguish variations in proficiency at the top of the distribution but not at the bottom, and conversely for tests with only easy items.

As seen, average pupils in SACMEQ countries fall in a score range where the TIMSS test provides quite little information. This major caveat must be kept in mind when comparing the estimates from various methodologies or attempting detailed analysis of scores from the African sample. This simple point also highlights a core challenge of international testing regimes: tests designed to provide information about OECD students may perform poorly in low-income countries and vice versa.

[12]I'm grateful to Abhijeet Singh for this suggestion.

6 Putting African mathematics scores in context

Whichever linking measure is chosen, math scores in sub-Saharan Africa are clearly very low relative to countries participating in international assessments. On average using the mean-sigma estimates from the previous section, African countries in my sample score roughly 300 points on the TIMSS scale, compared to approximately 488 for the rest of the sample. Is this higher or lower than would be expected given the region's generally low level of economic development?

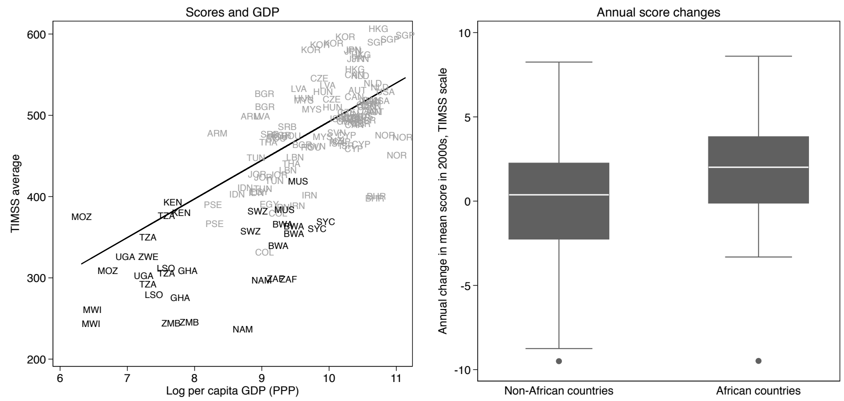
For almost all countries and years, the sub-Saharan African countries in either the SACMEQ or TIMSS data fall below the regression line of scores on GDP, as seen in Figure 9. In a simple regression of national mean scores on log per capita GDP in 2011 PPP dollars, scores rise roughly 35 to 40 TIMSS points with each log point of GDP. But Africa's low GDP is insufficient to explain its low test scores. Controlling for both log GDP and net primary enrollment rates, the coefficient on the dummy variable for Africa is negative and significant using all three linking methodologies, and ranges from approximately two-thirds to one and a half standard deviations. (See Table 5, columns 2, 4, and 6.)

Up to this point I have focused on score levels at a point in time, and focused the linking analysis on the 2000 SACMEQ round which contained TIMSS items. The SACMEQ test also enables trend analysis on the TIMSS scale. The majority of math items in the 2000 SACMEQ round were maintained in the 2007 instrument, and I include this 2007 item-level data in the estimation of the IRT model from equation (1) to yield scores for both rounds that are automatically on a comparable footing.

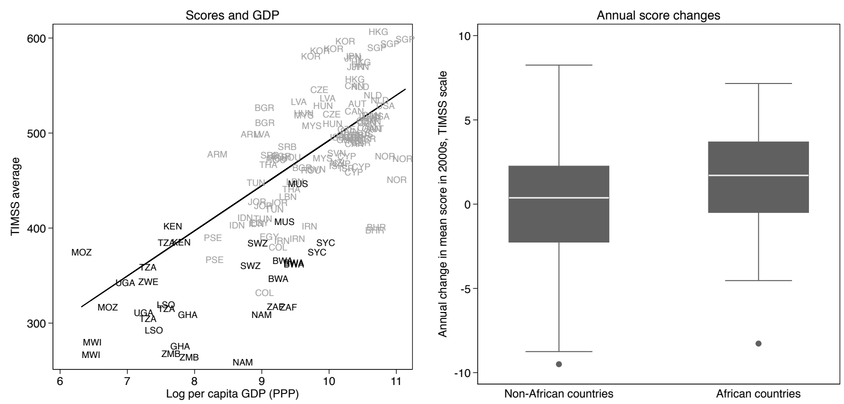
Is there evidence that African mathematics scores are converging to the international average? The answer is tentatively, no, or at best only slightly. The box plots in Figure 9 show annualized changes of country averages for the African and non-African countries in

Figure 9: Score levels and changes over time: African and non-African countries

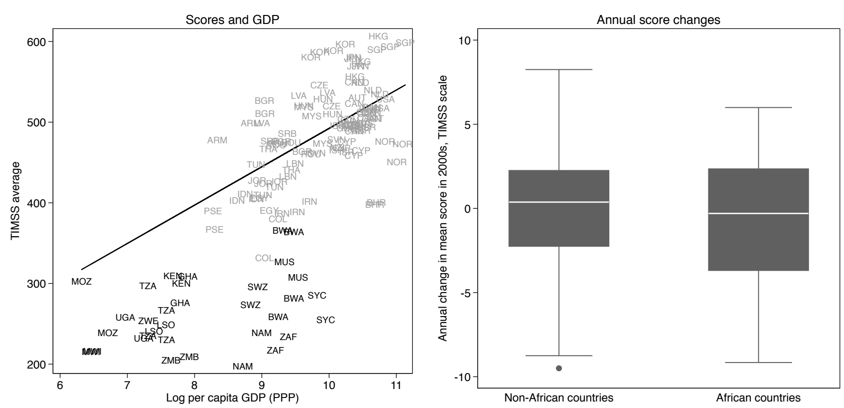
(a) Equipercentile linking



(b) Fixed parameter IRT linking



(c) Mean-sigma IRT linking



Note: In the scatter plots, African countries are shown in black and non-African countries in grey. The regression lines are fitted using only the non-African sample. The box plots show the average annual change during the 2000s. For the SACMEQ countries, the changes span 2000 to 2007; for the TIMSS sample the changes span 2003 to 2007.

Table 5: Are African mathematics scores lower than other development indicators predict?

	Equipercentile		Fixed parameter		Mean-sigma	
	(1)	(2)	(3)	(4)	(5)	(6)
Africa = 1	-155.8***	-79.20***	-142.8***	-65.25***	-224.0***	-153.4***
	(16.49)	(22.43)	(16.01)	(22.00)	(13.61)	(22.56)
Log per capita GDP (2011 PPP)		38.15***		38.10***		35.93***
		(8.000)		(7.853)		(7.928)
Net primary enrollment		0.605		0.684		0.451
		(0.607)		(0.594)		(0.621)
Year (2000 = 0)	-1.902**	-2.260***	-1.932**	-2.312***	-2.172***	-2.476***
	(0.791)	(0.745)	(0.789)	(0.737)	(0.788)	(0.751)
Obs.	137	137	137	137	137	137
R-squared	0.53	0.68	0.49	0.66	0.71	0.80

Note: Each column reports a separate linear regression of average scores in a given country and year on the TIMSS grade 7 and 8 scale. The dependent variable is measured with three different equating methods to link SACMEQ scores to the TIMSS scale: equipercentile linking (columns 1-2), fixed parameter IRT linking (3-4), mean-sigma IRT linking (5-6). All standard errors are clustered at the country level. Asterisks (*,** ,***) denote coefficients that are significantly different from zero at the 10, 5 and 1% levels, respectively.

the sample. (In all cases, the non-African results are based on original TIMSS data). From 2000 to 2007, mathematics scores in the original TIMSS sample were essentially unchanged, rising just 0.02 points per annum. Using both the fixed-parameter IRT methodology and the equipercentile linking methodology, the African countries reported increases of at least one point per annum. This does not hold for the mean-sigma linking, however, which shows a decline of roughly 0.7 points per annum. Notably, none of these differences are statistically significant, and the absolute rate of change for the African countries is very slow using any methodology. Given the score gaps reported in Table 5, the most optimistic results would still imply several decades if not centuries are needed for African countries' mathematics performance to converge to OECD levels at current rates of progress.

7 Conclusion

The sparse coverage of developing countries by international learning assessments creates a need for linking or equating methods to compare learning scales between regional and international assessments. This paper presents the results of applying three different equating methods to mathematics scores from the international TIMSS program and the SACMEQ test administered in Southern and Eastern Africa. The first equating approach relies on the overlap in coverage, due to South Africa and Botswana's participation in both tests. The other two equating methods rely on the inclusion of TIMSS items in the SACMEQ test, applying item-response theory methods to equate scores.

Mathematics scores in the African sample are low, both in absolute terms and relative to the region's economic development. These comparisons must be interpreted with the important caveat that sixth-grade pupils taking the SACMEQ test are two full grades behind most of their peers taking the TIMSS assessment in grade eight, although they are of similar ages due to over-age enrollment in the African sample. Furthermore, trends during the 2000s do not suggest African countries were catching up with the rest of the global sample, despite (or perhaps because of) significant gains in student enrollment over this period.

A key finding from this exercise is its limitations. The three equating approaches produce somewhat different results: while these estimates scores are highly correlated, their overall levels differ significantly. At a technical level, two reasons for the non-robustness of the results stand out. First, many of the linking items used in the formal IRT analysis exhibit high levels of differential-item functioning between the SACMEQ pupil and teacher samples (the teacher test provides a necessary bridge to the TIMSS test in this case). There is some evidence this is linked to the high difficulty of the TIMSS items for the SACMEQ pupil sample; DIF is much lower for the SACMEQ teachers, and for the TIMSS grade 3 and 4 items included on the SACMEQ test. Second, the number of linking items is simply quite small, and there is reason to believe that linking errors may become quite large when comparing populations with very different proficiency levels (e.g., Malawi and Canada) on the basis of relatively few items.

The implication for the design of future regional and international learning assessments is clear. If policymakers aim to make broad international comparisons, considerably greater care must be taken at the design stage. In order to link independent tests ex post, the number of linking items required may be closer to a few dozen or more, rather than the mere handful used in many applications at present. Crucially, the difficulty and discrimination of those items must be suited to the population in question. The TIMSS test, for instance, simply contains quite little information on pupils with the overall proficiency levels seen in many countries in Southern and Eastern Africa.

Future research should aim to overcome these deficiencies in test design, fielding instruments that include more anchor items overall, and ensuring that those items are appropriate to the population's academic proficiency level. If, with such data in hand, researchers can show that the various linking methods presented here generate congruent results, policymakers might then be justified in putting much greater faith in global learning metrics.

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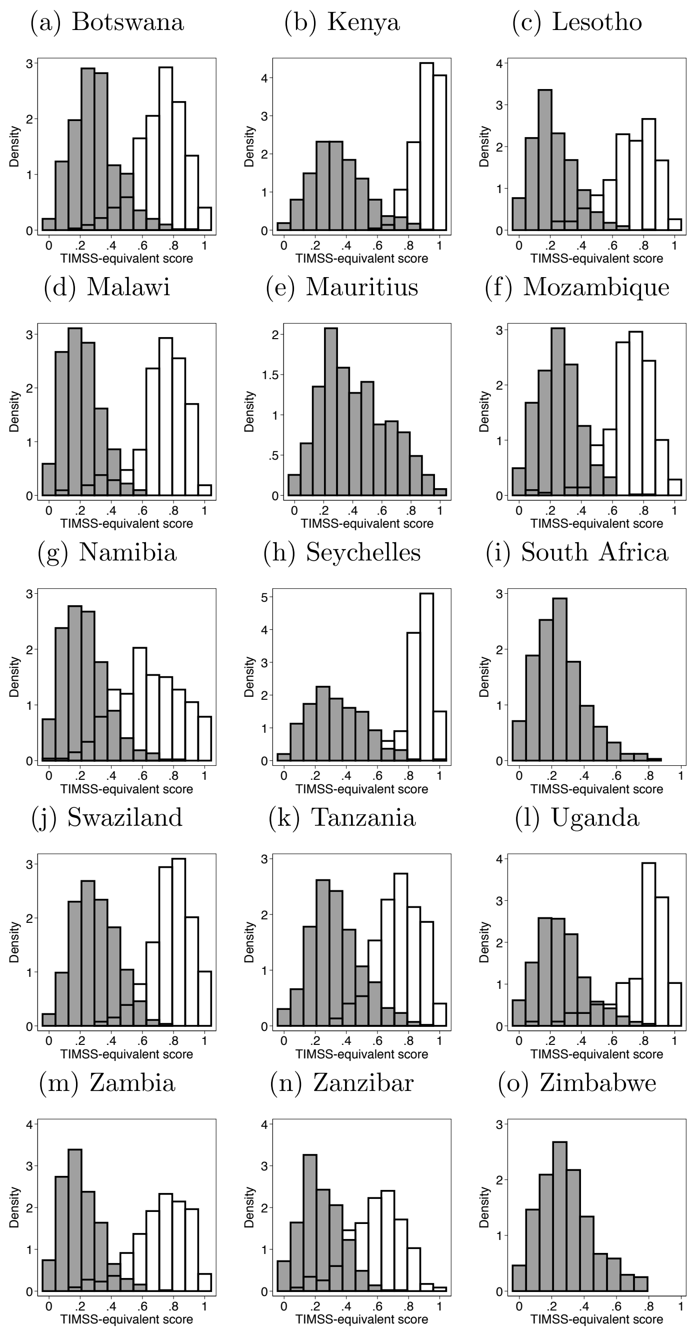
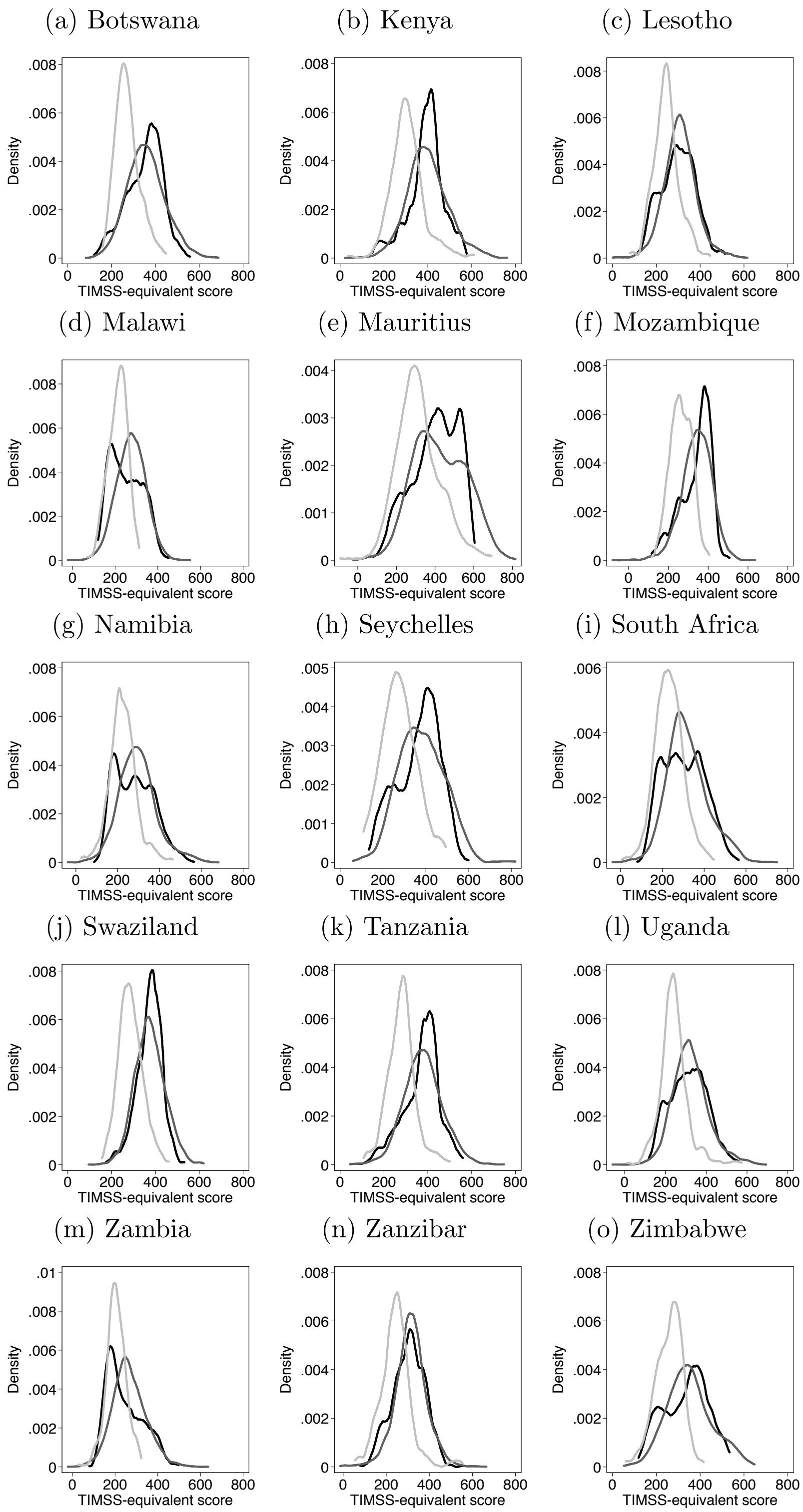
Appendix A: Additional figures and tables

Figure 10: Histograms of percent correct by teachers and pupils on overlapping items, by country



Note: Shaded bars show the distribution of pupil percent correct; hollow bars show the distribution for teachers on the same set of items.

Figure 11: Kernel density of TIMSS-equivalent pupil scores, by country



Note: Black lines are based on the equipercentile method; dark gray lines on the fixed parameter IRT linking; and light gray lines on the mean-sigma IRT linking.

Table 6: Differential Item Functioning: Mantel-Haenszel Tests

	SACMEQ pupils & teachers		TIMSS & SACMEQ pupils	
	Odds Ratio	χ^2 P-Value	Odds Ratio	χ^2 P-Value
Pupil item 26	0.5	0.00	.	.
Pupil item 27	3.3	0.00	1.1	0.00
Pupil item 28	0.7	0.00	.	.
Pupil item 29	0.5	0.00	.	.
Pupil item 30	0.9	0.19	.	.
Pupil item 32	1.7	0.00	.	.
Pupil item 33	1.4	0.00	.	.
Pupil item 44	1.3	0.00	.	.
Pupil item 55	1.1	0.13	0.5	0.00
Pupil item 56	3.0	0.00	2.0	0.00
Pupil item 57	1.5	0.00	.	.
Pupil item 58	0.8	0.04	.	.
Pupil item 63	0.5	0.00	.	.

P-values are reported for a χ^2 test for an odds ratio of unity, i.e., no DIF. Note that it is only possible to perform the MH test for three items in the TIMSS and SACMEQ pupil sample (right-hand panel) because the other anchoring items are spread across multiple booklets in the TIMSS data and are never administered to the same student.

Appendix B: Linking to TIMSS grade 3 and 4 scale

The SACMEQ test contained items from both the TIMSS grade 3 and 4 test (population 1, in TIMSS parlance) as well as the grade 7 and 8 test (population 2). The two tests use separate scales. In the main text, I focus on linking SACMEQ scores to the TIMSS grade 7 and 8 scale. While SACMEQ pupils and TIMSS grade 7 & 8 pupils are roughly the same age, SACMEQ pupils are on average four years older than the pupils who took the TIMSS grade 3 & 4 test (Table 1), so comparing them is somewhat artificial.[13]

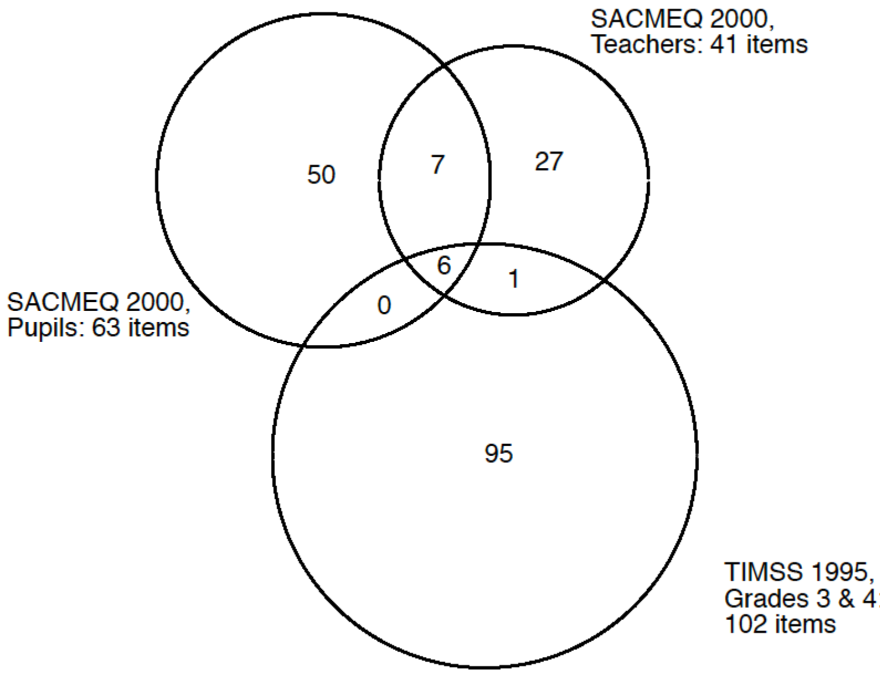
Nevertheless, for completeness, this section presents the link from SACMEQ scores to the TIMSS grade 3 and 4 scale. One major advantage of this link over the upper grades used in the main text is that it is direct: there are six overlapping items between the TIMSS pupil and SACMEQ pupil tests, allowing me to avoid passing through the SACMEQ teacher test as in the main text. (See Figure 12.) Furthermore, identifiers exist to enable me to link the item-level TIMSS data to the corresponding items in SACMEQ, which is not possible for the upper grades.

The African countries in my sample perform surprisingly well on the six items that overlap from the SACMEQ pupil test and TIMSS grade 3 and 4 test. While it is true that African countries occupy the bottom spots in the country ranking in Figure 13, it is interesting that Kenya performs better than the United States – bearing in mind, again, that the pupils in the Kenyan sample are roughly four years older than their American counterparts.

Notably in Figure 14 , the degree of differential item functioning (DIF) for the items taken from the TIMSS lower-grade scale appears relatively low compared to what was observed for the items taken from the upper-grade scale. This is consistent with the hypothesis that the

[13] Additional motivation for focusing on the upper-grade scale is that there are fewer linking items at the lower grades, and documentation does not exist to link the lower-grade items from the 1995 TIMSS test to the three-parameter IRT model adopted in subsequent years – rendering the results, even if successfully rescaled on the TIMSS 1995 scale, incomparable to more recent TIMSS scores.

Figure 12: Heuristic of overlapping math items



The Venn diagram shows the overlapping or "anchor" items linking the SACMEQ 2000 pupil and teacher tests to the TIMSS 1995 test of third- and fourth-graders.

TIMSS upper-grade items were simply too difficult for the SACMEQ sample and resulted in arbitrary guessing, while the lower-grade items may have been more appropriate.

In contrast, the variance of the TIMSS-scaled scores for the SACMEQ countries appears quite low in Figure 15. The range from the fifth to the ninety-fifth percentile is much narrower for the African sample than the original TIMSS countries – exactly the opposite of the pattern found for the upper-grade scale in the main text. This may be an artifact of the low number of anchoring items used to link the lower-grade scale. The possible variance is simply lower with only a handful of items.

Figure 13: Country rankings: Percent correct, overlapping TIMSS grade 4 items

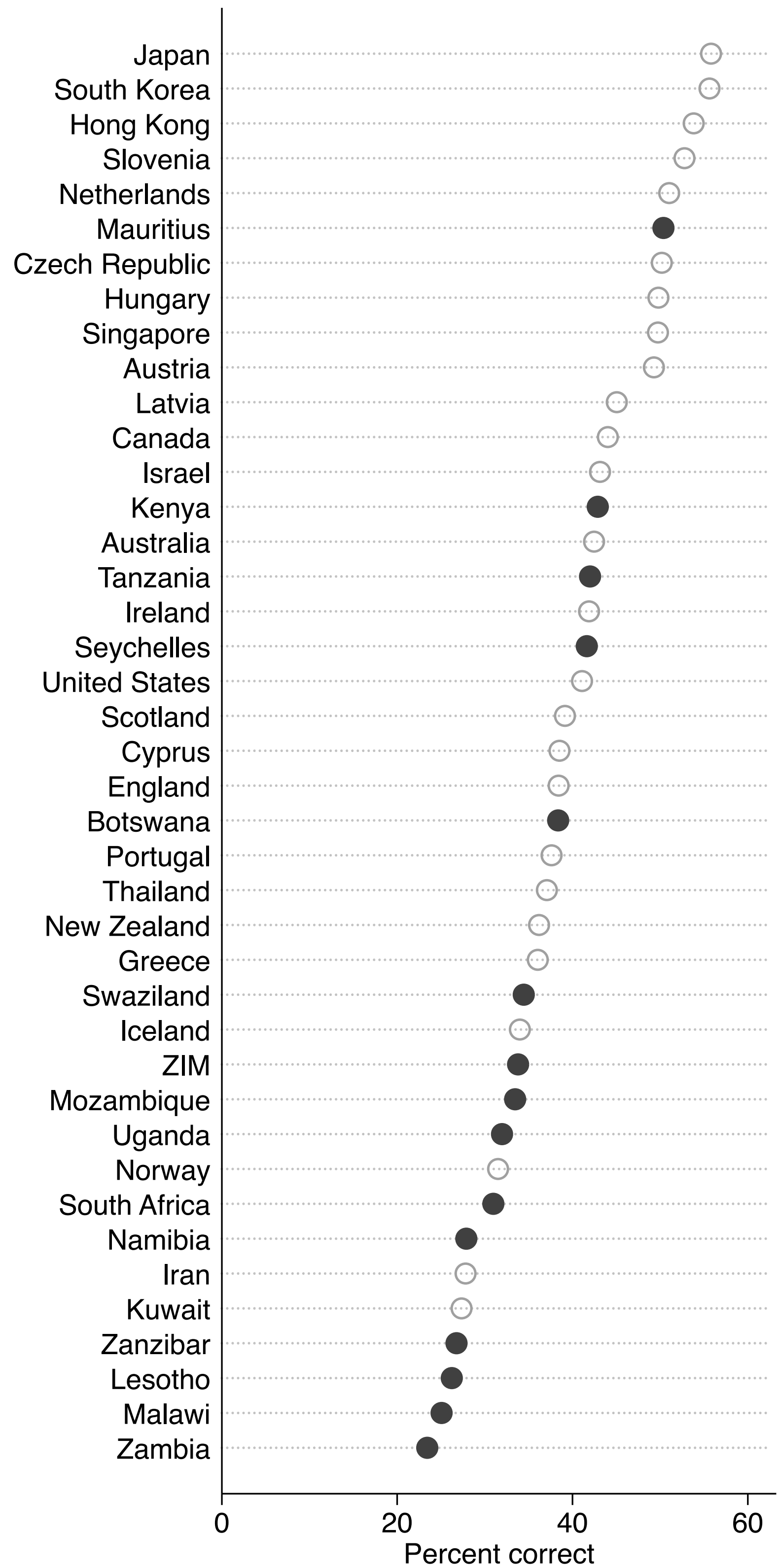
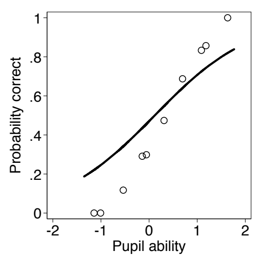
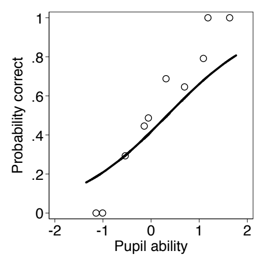


Figure 14: DIF: SACMEQ pupils vs. IRF from TIMSS 4th-graders

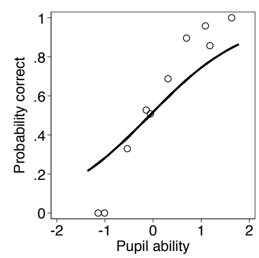
(a) Item 27



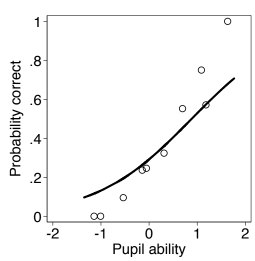
(b) Item 28



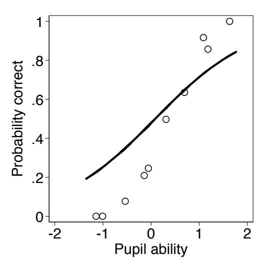
(c) Item 29



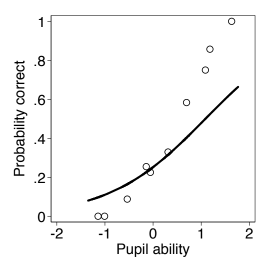
(d) Item 30



(e) Item 56



(f) Item 57



Note: Dots represent the observed percent correct for SACMEQ pupils. Lines show the predicted percent correct based on the estimated IRF for TIMSS pupils in grades 3 and 4.

Figure 15: Country rankings: TIMSS grade 4 scale, IRT linking